Formulaire : développements limités en 0 des fonctions usuelles

Formule 1

$$\frac{1}{1-x} \underset{x\to 0}{=} \sum_{k=0}^{n} x^k + o(x^n) \underset{x\to 0}{=} 1 + x + x^2 + \cdots + x^n + o(x^n)$$

Formule 2

$$\frac{1}{1+x} \underset{x\to 0}{=} \sum_{k=0}^{n} (-1)^k x^k + o(x^n) \underset{x\to 0}{=} 1 - x + x^2 + \cdots + (-1)^n x^n + o(x^n)$$

Formule 3

$$e^x \underset{x\to 0}{=} \sum_{k=0}^{n} \frac{x^k}{k!} + o(x^n) \underset{x\to 0}{=} 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + o(x^n)$$

Formule 4

$$\cos x \underset{x\to 0}{=} \sum_{k=0}^{n} (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n+1}) \underset{x\to 0}{=} 1 - \frac{x^2}{2} + \frac{x^4}{24} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

Formule 5

$$\sin x \underset{x\to 0}{=} \sum_{k=0}^{n} (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) \underset{x\to 0}{=} x - \frac{x^3}{6} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

Formule 6

$$(1+x)^\alpha \underset{x\to 0}{=} 1 + \sum_{k=1}^{n} \alpha(\alpha-1)\dots(\alpha-k+1)\frac{x^k}{k!} + o(x^n) \underset{x\to 0}{=} 1 + \alpha x + \alpha(\alpha-1)\frac{x^2}{2} + \cdots + \alpha(\alpha-1)\dots(\alpha-n+1)\frac{x^n}{n!} + o(x^n)$$

Formule 7

$$\operatorname{ch} x \underset{x\to 0}{=} \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!} + o(x^{2n+1}) \underset{x\to 0}{=} 1 + \frac{x^2}{2} + \frac{x^4}{24} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

Formule 8

$$\operatorname{sh} x \underset{x\to 0}{=} \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) \underset{x\to 0}{=} x + \frac{x^3}{6} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

Formule 9

$$\tan x \underset{x\to 0}{=} x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^6)$$

Formule 10

$$\ln(1+x) \underset{x\to 0}{=} \sum_{k=1}^{n} (-1)^{k-1}\frac{x^k}{k} + o(x^n) \underset{x\to 0}{=} x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{n-1}\frac{x^n}{n} + o(x^n)$$

Formule 11

$$\ln(1-x) \underset{x\to 0}{=} -\sum_{k=1}^{n} \frac{x^k}{k} + o(x^n) \underset{x\to 0}{=} -x - \frac{x^2}{2} - \frac{x^3}{3} + \cdots - \frac{x^n}{n} + o(x^n)$$

Formule 12

$$\operatorname{Arctan} x \underset{x\to 0}{=} \sum_{k=0}^{n} (-1)^k\frac{x^{2k+1}}{2k+1} + o(x^{2n+2}) \underset{x\to 0}{=} x - \frac{x^3}{3} + \cdots + (-1)^n\frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$$