

Formulaire :

développements limités en 0 des fonctions usuelles

Formule 1

$$\begin{aligned}\frac{1}{1-x} &= \sum_{k=0}^n x^k + o(x^n) \\ &= 1 + x + x^2 + \cdots + x^n + o(x^n)\end{aligned}$$

Formule 2

$$\begin{aligned}\frac{1}{1+x} &= \sum_{k=0}^n (-1)^k x^k + o(x^n) \\ &= 1 - x + x^2 - \cdots + (-1)^n x^n + o(x^n)\end{aligned}$$

Formule 3

$$\begin{aligned}e^x &= \sum_{k=0}^n \frac{x^k}{k!} + o(x^n) \\ &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + o(x^n)\end{aligned}$$

Formule 4

$$\begin{aligned}\cos x &= \sum_{k=0}^n (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n+1}) \\ &= 1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})\end{aligned}$$

Formule 5

$$\begin{aligned}\sin x &= \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) \\ &= x - \frac{x^3}{6} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})\end{aligned}$$

Formule 6

$$\begin{aligned}(1+x)^\alpha &= 1 + \sum_{k=1}^n \alpha(\alpha-1)\cdots(\alpha-k+1) \frac{x^k}{k!} + o(x^n) \\ &= 1 + \alpha x + \alpha(\alpha-1) \frac{x^2}{2} + \cdots + \alpha(\alpha-1)\cdots(\alpha-n+1) \frac{x^n}{n!} + o(x^n)\end{aligned}$$

Formule 7

$$\begin{aligned}\operatorname{ch} x &= \sum_{k=0}^n \frac{x^{2k}}{(2k)!} + o(x^{2n+1}) \\ &= 1 + \frac{x^2}{2} + \frac{x^4}{24} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1})\end{aligned}$$

Formule 8

$$\begin{aligned}\operatorname{sh} x &= \sum_{k=0}^n \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2}) \\ &= x + \frac{x^3}{6} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})\end{aligned}$$

Formule 9

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^6)$$

Formule 10

$$\begin{aligned}\ln(1+x) &= \sum_{k=1}^n (-1)^{k-1} \frac{x^k}{k} + o(x^n) \\ &= x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{n-1} \frac{x^n}{n} + o(x^n)\end{aligned}$$

Formule 11

$$\begin{aligned}\ln(1-x) &= -\sum_{k=1}^n \frac{x^k}{k} + o(x^n) \\ &= -x - \frac{x^2}{2} - \frac{x^3}{3} + \cdots - \frac{x^n}{n} + o(x^n)\end{aligned}$$

Formule 12

$$\begin{aligned}\operatorname{Arctan} x &= \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{2k+1} + o(x^{2n+2}) \\ &= x - \frac{x^3}{3} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})\end{aligned}$$